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# Folded Catadioptric Cameras

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Many applications in computational vision, such as surveillance or teleconferencing, require the imaging of a large field of view. Unfortunately, conventional imaging systems, like video cameras, are severely limited in their fields of view. To image an entire scene, either multiple cameras or a single rotating camera had to be used in the past. A modern and effective way to enhance the field of view is to use *catadioptric systems*[1] — mirrors in conjunction with lenses.

## 1 Traditional Imaging Systems

Most of today's imaging systems consist of a video (or photographic) camera attached to a lens. The image projection model for most camera lenses is perspective with a single center of projection. The field of view of a camera lens typically corresponds to a small cone rather than to a hemisphere (see Figure 1).

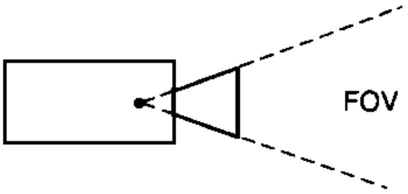

Figure 1: Field of view of a typical camera lens

The following sections describe several approaches to achieve a wider field of view.

### 1.1 Rotating Imaging Systems

An obvious solution to obtain a wider field of view is to rotate the entire imaging system about its center of projection. To get a panoramic view of the entire scene, the individual images are placed together after

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[1] The science of refracting elements (lenses) is called *dioptrics* and the science of reflecting elements (mirrors) *catoptrics*; therefore the combination of both refracting and reflecting elements is referred to as *catadioptrics*.

the capturing has been completed.

A disadvantage of rotating imaging systems is that they require the use of moving parts which have to be positioned very precisely. Another serious drawback of these systems is the total time required to obtain an image with a wide field of view. This restricts the use of such systems to static scenes and non-realtime applications.

## 1.2 Fish-Eye Lenses

Another approach to obtain images with an enhanced field of view is the use of a fish-eye lens in place of a conventional camera lens. Such lenses have a very short focal length and thus enable cameras to view objects within a nearly hemispherical area.

The serious disadvantage of this approach is that it is very difficult to design a fish-eye lens that has a fixed effective viewpoint (see section 2.1 for an explanation why this is desirable). Therefore it is impossible to construct distortion-free perspective images from the captured scene. Additionally, to get a full hemispherical view, the fish-eye lens must be quite complex and large, and hence expensive.

## 1.3 Catadioptric Systems

Catadioptric imaging systems use a reflecting surface – typically a mirror – to achieve a wider field of view. A common example for such a system is the rear-mirror in a car.

However, the shape, position and orientation of the reflecting surface is related to the viewpoint and the field of view in a complex manner. While it is easy to significantly increase the field of view with a catadioptric system, it is hard to keep the effective viewpoint fixed in space.

# 2 Design Criteria for Single-Mirror Catadioptric Systems

## 2.1 The Fixed Viewpoint Constraint

A catadioptric imaging system uses a combination of lenses and mirrors placed in a carefully arranged configuration to capture a much wider field of view than conventional imaging systems.

In [Baker and Nayar, 1998] the authors explain why, when designing a catadioptric sensor, the shape of the mirror should ideally be selected to ensure that the system has a *single effective viewpoint* (center of projection). The reason a single viewpoint is so desirable is that the construction of geometrically correct and distortion-free perspective images from an image captured by a catadioptric camera is possible only when the “fixed viewpoint constraint” is fulfilled. Also, when presenting the images to a human, they have to be in perspective so as not appear distorted.

In [Baker and Nayar, 1998] the entire class of catadioptric systems which are constructed using a conventional lens and a single mirror and which have a single effective viewpoint is derived. Furthermore, a general solution of the “fixed viewpoint constraint” as well as specific solutions for different mirror shapes are given. The solutions reveal that, to ensure a fixed viewpoint, the mirror must have a planar, elliptic, hyperbolic or parabolic shape.

## 2.2 Resolution

An important property of a catadioptric system which images a large field of view is its resolution. In [Baker and Nayar, 1998] it is shown why the total resolution of a system is, in general, not the same as that of the individual sensors used to construct it. The authors derive an expression for the relationship between the resolution of a conventional imaging system and the resolution of a derived catadioptric sensor which should be carefully considered when constructing a catadioptric imaging system in order to ensure that the final sensor has sufficent resolution.

### 2.3 Focusing

Another optical property which is modified by the use of a catadioptric system in place of a conventional imaging system is focusing. In conventional imaging systems, there are two main factors which cause defocus blur: diffraction and lens aberrations. When using a catadioptric camera, two additional factors combine to cause further blur: the finite size of the lens aperture and the curvature of the mirror. [Baker and Nayar, 1998] includes an analysis of the interaction of these two factors which shows that the focal settings of a catadioptric sensor using a curved mirror may substantially differ from the ones needed with a conventional sensor.

## 3 Folded Catadioptric Systems

### 3.1 What Is a Folded System?

A major problem with catadioptric imaging systems is that they tend to be physically large when compared to conventional ones. This is due to the fact that the capture of a wide unobstructed field of view requires the lens and the mirror to be adequately separated from each other.

By using multiple mirrors within a catadioptric system, the optics can be *folded* and thus more compact camera designs can be achieved. A simple example is the use of a planar mirror to fold the optical path between a curved mirror and an imaging lens. Folding by means of a curved mirror can result in even greater size reduction. More importantly, curved folding mirrors can serve to reduce undesirable optical effects, such as field curvature, caused by a curved primary mirror.

[Nayar and Peri, 1999] presents several camera designs which use two conic mirrors and shows that any folded system with two conic mirrors has a *geometrically equivalent* system that uses a single conic mirror. Even if *geometric* equivalence does *not* imply *optical* equivalence, it is valuable in that it can be used to determine the relation between scene points and image coordinates, which is needed to construct perspective or panoramic images from scenes captured by a folded system.

### 3.2 The General Problem of Folding

The general problem of designing folded imaging systems is to determine, for a given desired viewpoint and a desired field of view, the shapes, positions and orientations of the mirrors that would reflect the entire scene through a single point (the center of projection of the imaging lens).

[Nayar and Peri, 1999] presents an elegant method for determining the shape of the secondary mirror that maps the scene rays in direction of a chosen viewpoint to a chosen imaging point for a primary mirror of *arbitrary shape*.

Though with this method a variety of exotic mirror pairs can be found to construct a folded imaging system with a single viewpoint, typically only *conic* mirrors are used. Complex mirror shapes tend to produce several optical aberrations (see section 4.1) that cause image quality to vary dramatically over the field of view. Conic mirrors have a well-defined focus and it is therefore easy to combine conic mirrors in a way that ensures a fixed viewpoint.

# 4 Optics of Folded Systems

The geometric design of a folded system considers the "pupil" of the system to be only a small pinhole and thus takes into account only the principal rays which enter the "pupil" directly. If a lens is used to gather more light, each principal ray is accompanied by many surrounding rays which leads to several optical aberrations that makes the design of a folded system challenging.

## 4.1 Relevant Optical Effects

### 4.1.1 Chromatic Aberrations

The focal length of any lens varies with the "color" (the wavelength) of the incoming light which leads to chromatic aberrations (see Figure 2a).

An imaging lens consists of several individual elements and one of the design goals for such lenses is to ensure that chromatic aberrations induced by the individual elements at least partially compensate for each other.

### 4.1.2 Coma and Astigmatism

Coma[2] and astigmatism[3] are caused primarily due to the curvature of the mirrors. They both cause the best focused image of a scene point to not be a single point but rather a volume (see Figure 2(b) and (c)).

The effect of coma is proportional to the square of the aperture size, while astigmatism is linear in the aperture size. The design goal for a folded system is to maximize the aperture size while ensuring that the blur function (the aberrations caused by coma and astigmatism) falls within a single pixel for all points in the field of view.

### 4.1.3 Field Curvature

Rays reflected by a curved mirror are best focused not on a plane but rather a curved surface behind the imaging lens, which is also called *Petzval Surface* (see [Hecht and Zajac, 1974]).

When using planar CCD (Charge Coupled Device) imagers, the best image quality is achieved where the curved image and the planar detectors intersect. In compact systems, where small mirrors with high curvatures are used, the field curvature tends to dominate over all other aberrations.

In a single-mirror system, the image surface is curved in the same direction as the mirror itself. Hence, in a two-mirror system a convex and a concave mirror can be

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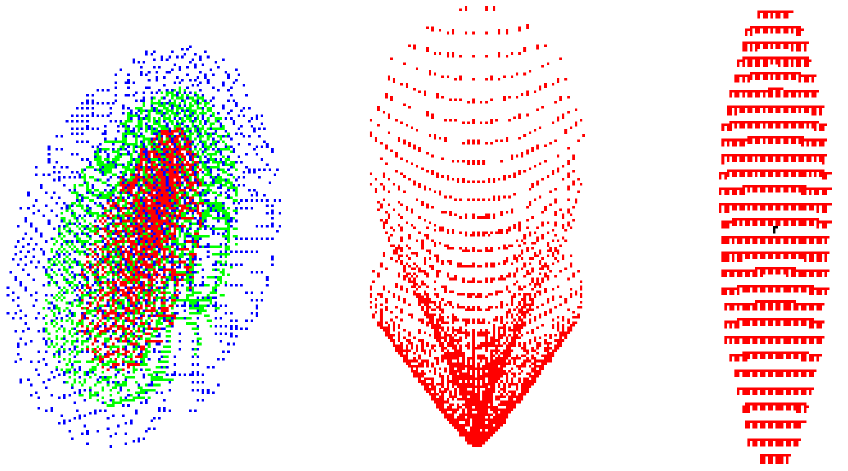
[2] If light goes through a lens off axis (at an angle) the light will not focus to a single point but look like a fuzzy circle. The farther off axis, the larger the circle, giving the images a comet-like look; hence the name "coma".

[3] Defect of vision due to the radius of curvature of the optics being unequal at different orientations around the visual axis. Lines or bars at different orientations are not all simultaneously in focus, and there can be distortions for some orientations.



(a) Chromatic aberr. (b) Coma (c) Astigmatism

Figure 2: (a) Chromatic aberrations due to different focal lengths dependent on the "color" of the incoming light. (b) Coma looks like a comet. (c) Astigmatism shows X/Y-axis asymmetry.

used so that the field curvature introduced by both mirrors compensate for each other.

## 4.2 Design Parameters

The design of a catadioptric system requires the careful selection of optical parameters to minimize the aberrations described above. Therefore the following parameters should be taken into account.

### 4.2.1 CCD Size

There are a few different CCD formats commercially available (1 inch, 1/2 inch, 1/3 inch, etc.). While the *number of pixels* in each CCD is more or less the same, the *pixel size* reduces with CCD size. The choice of the CCD format typically depends on the packaging and resolution requirements of the application.

### 4.2.2 Imaging Lens

The parameters of an imaging lens are characterized by its focal length, field of view, aperture size and the number of elements. While the number of elements and the basic shape (convex, concave, etc.) of the lens may be selected up front, the curvature and diameter may be treated as free parameters which are computed during the phase of system optimization. Once the optimization is finished, one tries to match the resulting parameters with those of commercially available lenses.

### 4.2.3 Mirrors

As mentioned in section 3.2, a large number of mirror shapes can be used in a folded catadioptric system. When designing such a system, the general shapes of the mirrors to be used must be selected up front based on the desired size and field of view of the system, as well as a good deal of intuition. Since it is known that the use of a convex and a concave mirror helps to reduce field curvature (see section 4.1.3), it might be a good choice to use such a combination. After the general shapes have been chosen, the exact shape parameters can be treated as free parameters for the optimization.

### 4.2.4 Distances

To achieve a single viewpoint, the far focus of the primary mirror must coincide with the near focus of the second mirror. Additionally, due to the typically limited size of the whole system, the distances between the individual optical components are limited to fairly tight bounds. The exact parameters can be treated as free parameters during the stage of optimization.

# 5 An Exemplary Implementation

This section presents an exemplary implementation of a folded catadioptric camera (taken from [Nayar and Peri, 1999]).

## 5.1 The Device

Figure 3 shows the prototypical implementation of a folded panoramic video camera with hemispherical field of view. The device is 90 mm tall and 50 mm wide. It includes folded optics, a video camera and a microphone.

Figure 4 shows the layout for the device. It uses two parabolic mirrors, where the focal length of the secondary mirror is significantly longer than that of the primary one. This is because the two mirrors must be adequately separated to avoid a large blindspot.

Prior to the optimization of the system, it was decided that the complete device should fit into a cylinder of 90 mm in height and 50 mm in width. The desired field of view was set to a hemisphere and the maximum allowed blindspot to 22 degrees when measured from the optical axis. Additionally, it was decided that a 1/3 inch CCD camera would be used.

Given these constraints, the secondary mirror ends up being a small, shallow section of a paraboloid, which is well approximated by a spherical mirror. Using the given numbers as upper bounds, the parameters of the entire system were optimized.

## 5.2 Example Images

Figure 5(a) shows a hemispherical image captured by the camera presented in the section before. As it can be seen, despite all the complex optical aberrations which can occur, the camera produces a clear image over the complete field of view.

Figures 5(b) and (c) show perspective and panoramic images which are computed from the hemispherical image. The jagged artifacts are due to the low resolution (640x480) of the original image.

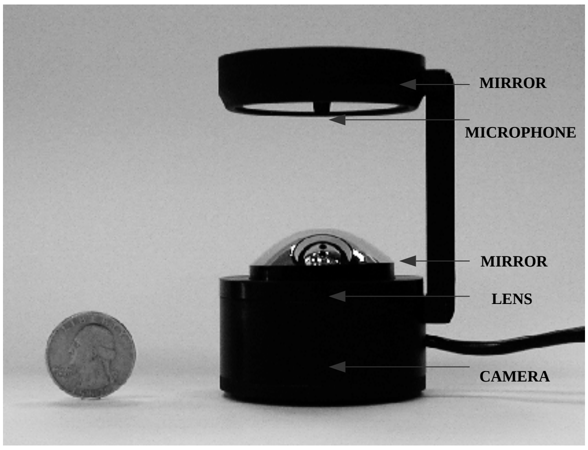


Figure 3: An exemplary implementation of a folded catadioptric camera

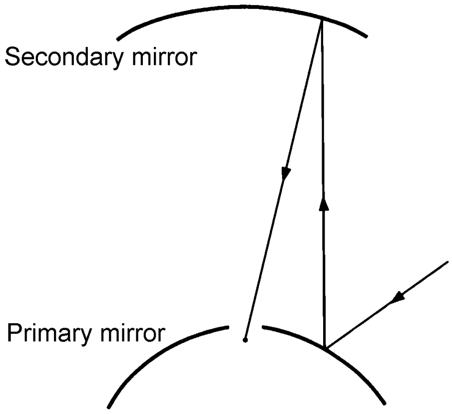
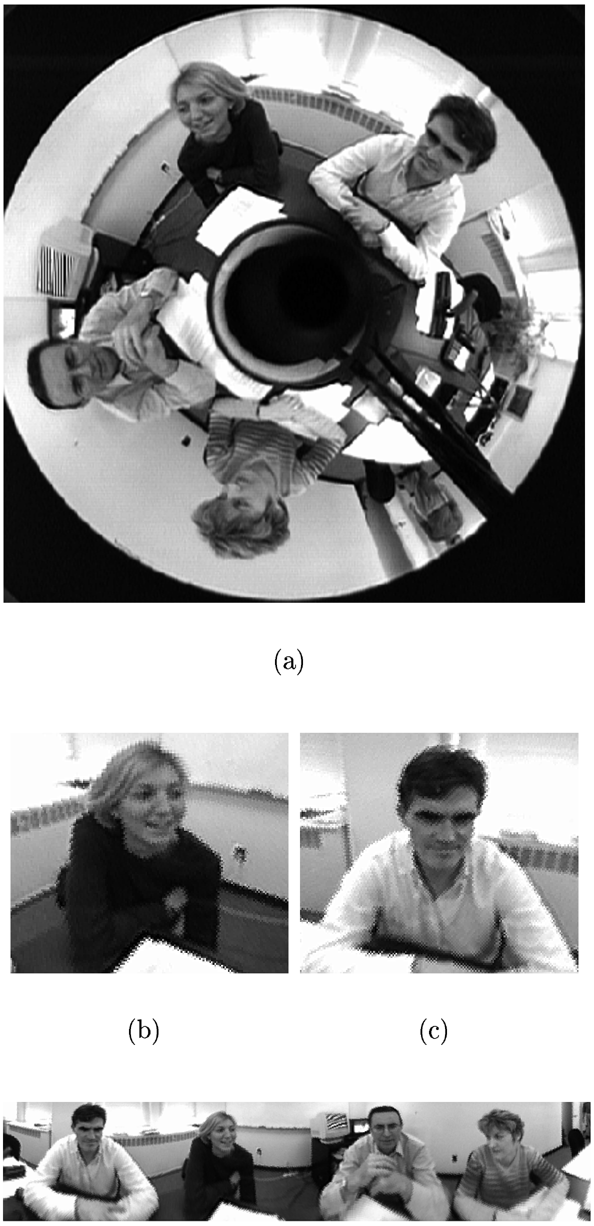


Figure 4: Layout of the camera


(a)

(b) (c)

(d)

Figure 5: (a) Captured hemispherical image. (b), (c) Computed perspective and (d) computed panoramic images.

# References

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